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DEGRADING MATH SKILLS: IT IS SUBJECT MATTER DIFFICULTY, NOT THE PASSAGE OF TIME, THAT MATTERS

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ABSTRACT

While the mediating factors have been the subject of intensive research over the years, it has long been assumed that what we know decays over time. Much research has focused on the decay of academic knowledge for subjects like math and reading. The purpose of the present research was to quantify the loss of mathematics skills over a long period of time in elementary, middle school and high school students. Three comprehensive mathematics tests were constructed: one that covers 5th grade math, one that covers Algebra 1 and one that covers Algebra 2. 31 6th graders, 17 students enrolled in geometry (the course after Algebra 1), and 15 students in precalculus (the course after Algebra 2) participated in the present study. The 6th graders were given the 5th grade math assessment, the geometry students were given both the 5th grade math and Algebra 1 assessments, and the precalculus students were given all three assessments. Results showed virtually identical performance across students of different math classes when taking the same test and a linear decline in performance for older students taking multiple assessments such that they fared worse on the more recent material than they did on the older material, going against traditional human memory findings. Students at all course levels averaged in the low to mid 80's in the 5th grade math assessment, geometry and precalculus students averaged in the mid 60's in the Algebra 1 assessment, while the precalculus students averaged in the mid 40's in the Algebra 2 assessment. Results suggest that subject matter difficulty rather than substantial passage of time accounts for decline in problem solving performance.

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Introduction

Ever since Thorndike posited his memory decay theory over 100 years ago (Thorndike, 1914), it has generally been assumed that memories decay over time. While many have debated the mechanism behind why this occurs, the consensus has been that, as time passes, people remember less and less of what they have learned or experienced. This principle applies to academic learning as well.

At MyEdMaster, we tutor students in academic subjects and test preparation. A common finding, particularly when students prepare for standardized tests that include material students learned years prior, is that most math errors seem to come from what students term “silly mistakes” rather than unlearned concepts. For lack of a better term, it seemed that students were getting “rusty” when attempting to solve problems using concepts they had not used frequently in recent years.

There have been some attempts to examine loss of academic skills over time. For example, Cooper et al. (1996) did a meta-analysis of 39 studies that showed loss of skills that occurred during students’ summer breaks. Based on their analysis, Cooper concluded that over summer break, students lost, on average, about one month’s worth of learning. Hill et al. (2007) focused on quantifying this summer loss by grade and subject and found that as students got older, the amount of summer loss decreased, with K-1 students’ performance dropping, on average by more than one standard deviation and 11th and 12th graders showing almost no drop in performance.

Other researchers have looked at more general trends in skill decline that are not dependent just on summer absence from school. For example, McCombs et al. (2011) report that absent practice, skills decay rapidly initially, but then stabilize over time. On the other hand, Arthur et al. (1998) found variable levels of skill decay over time with typical ranges around the one standard deviation level.

Both of the previous types of research on skill decay may not be adequate to address the observed patterns with our tutoring students. Since students receive tutoring year round and go to school for most of the year, any loss of skill would not be attributable solely to a summer lapse. Also, the skills that we observe being degraded were not just the ones taught in the previous school year, so any time interval that accounts for the skill loss would be far greater than just a summer. Similarly, sheer inactivity over time is not sufficient to explain loss of math skills. Math is a cumulative subject and concepts learned in early grades and along the way are repeated in successive math classes. Therefore, degradation of math skills cannot be explained solely by non-use.

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The present project seeks to quantify how much math skills degrade over time. While there seems to be strong consensus that math skills do degrade, we believe that there has been little attempt to quantify how much different math skills do degrade over time, apart from non-use during summer or other time period, and if there is any orderliness to the degradation process. In order to examine this question, we picked three grade levels of math: fifth grade math, Algebra 1 and Algebra 2.

Fifth grade math was chosen as one of the grades because in Fairfax County, Virginia, where MyEdMaster is located, 6th grade students are given a placement exam to see if they will be placed in advanced math. Algebra 1 was chosen since it is used in placement exams taken by 8th graders for admission to specialized STEM (science, technology, engineering, and math) high schools in Northern Virginia. Algebra 2 was chosen because it is the highest math used on the SAT test that high schoolers take as part of their college admissions process. It should also be noted that concepts used at each grade level form the foundation of the concepts taught in the higher grades.

Method

Participants

Participants were thirty-one sixth-graders, seventeen students who were taking geometry, and fifteen who were taking precalculus. Participants were recruited from schools in both the metropolitan Washington, DC area and in northern California.

Materials

Three mathematics assessments were created. One was an assessment covering 5th grade math, one was an assessment covering Algebra 1, and one was an assessment covering Algebra 2. All assessments were created based on topics contained in math textbooks created by leading American publishers.

The subjects for the fifth-grade assessment were: addition, subtraction, multiplication, division, digit placement, greater than/ less than/equal to, fractions, factors/multiples, angles, perimeter/area/volume, shapes, order of operations, measurement conversions, exponents, and decimals. The subjects for the Algebra 1 assessment were all of those included in the fifth-grade test, along with algebraic equations, systems of equations, slope, intercepts, functions/properties of functions, tables, direct/indirect variations, evaluating algebraic expressions, factoring, quadratic formula, and inequalities. The subjects for the Algebra 2 test were all questions in the Algebra 1 test, along with exponent/radical equations, sequences and series, complex factoring,

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imaginary Numbers, functions and lines, linear inequality, roots of functions, properties of functions, rational Expressions and equations, exponential growth and decay, trigonometry (including radians vs degrees), and probability/statistics.

Procedure

Each participant was given a course/grade-appropriate tests. The tests were administered in the fall and winter of 2020, which meant they were given in the semester immediately following the students' most recently completed math class, but not immediately after summer break. In other words, 6th graders received the 5th grade math assessment during the semester after they completed 5th grade math, geometry students received the Algebra 1 assessment during the semester after they completed Algebra 1, and precalculus students received the Algebra 2 assessment during the semester after they completed Algebra 2. All tests were administered online in a supervised environment. They were not allowed to use calculators or outside resources. Participants were given as much time as they need to complete the test.

Results

The results of each participants' assessment was scored based on total number of correct answers given, broken down by topic as described in the Method section.

Table 1: Mean Percentage of 5th Grade Problems Solved Broken Out by Problem Type and Participants' Current Math Level

Topics:	**6th Graders**	**Geometry students**	**Pre Calculus students**
Addition	90.3%a	98.04%a	100%a
Subtraction	88.17%a	94.12%a	100%a
Multiplication	72.5%	97.06%a	100%a
Division	80.64%a	88.24%a	81%a
Digit placement	80.64%a	85.29%a	95.4%a
Greater than/ Less than/Equal to	93.55%a	94.12%a	100%a
Fractions	81.05%a	91.18%a	74.5%a

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Factors/multiples	90.32%a	88.24%a	81.8%a
Angles	96.77%a	100%a	90.9%a
Area/volume	82.26%a	66.18%a	86.3%a
Perimeter	83.87%a	100%	72.7%a
Shapes	72.58%a	76.47%a	81.8%a
Order of operations	88.7%a	100%a	86.36%a
Measurement Conversions	65.8%a	69.41%a	72.7%a
Exponents	83.87%a	86.27%a	90.9%a
Decimals	91.93%a	97.06%a	90.9%a
Overall	81.5%a	84.5%a	84.1a

Note: percentage values not sharing a common subscript are statistically significantly different at the .05 level.

Table 2: Mean Percentage of Algebra 1 Problems Solved Broken Out by Problem Type and Participants' Current Math Level.

Topics	**Geometry Students**	**Pre Calculus Students**
Algebraic equations	94.12%	69.1%
Systems of equations	50%a	43.9%a
Slope	80.88%a	77.2%a
Intercepts	35.29%a	51.5%a
Functions/ properties of functions	74.26%a	61.04%a
Tables	64.71%	36.3%
Direct/indirect variations	38.24%a	45.45%a
evaluating algebraic expressions	81.45%a	80%a
Factoring	75%a	64.94%a

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Quadratic formula	58.82%a	66.67%a
Inequalities	80%a	56.36%a
Overall	65.6%a	65.5%a

Note: percentage values not sharing a common subscript are statistically significantly different at the .05 level.

Table 3: Mean Percentage of Algebra 2 Problems Solved by precalculus Students Broken Out by Problem Type.

Topics	**Pre Calculus Students**
Exponent/Radical Equations	27.2%
Sequences and Series	30.3%
Complex Factoring	36.4%
Imaginary Numbers	63.6%
Functions and lines	54.54%
Linear Inequality	72.7%
Roots of Functions	27.27%
Properties of Functions	36.36%
Rational Expressions and Equations	36.36%
Exponential Growth and Decay	36.36%
Trigonometry (and radians vs degrees)	58.04%
Probability/Statistics	27.27%
Overall	46.6%

If we look at the data, there seem to be three noteworthy trends. First, there seem to be very few topics for which students are, in general, truly proficient in. Arithmetic seems to be a noteworthy example. Even though the assessments covered topics that students have already studied and been tested on, there were several instances in which average performance was as low as in the 20% range. In fact, the average score for the Algebra 2 test results was 46.65, considered failing by typical grading standards. The average score for the Algebra 1 test results was in the D range

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for both geometry and precalculus students. This suggests that math skills had degraded significantly from the time students first learned the concepts until the time they took the present assessment. Moreover, the amount of degradation seemed much greater than what had previously been reported in the literature.

Second, contrary to what might be expected from the general literature on memory, passage of time appeared to have no effect on skill level. For the fifth grade math assessment, current sixth graders averaged 81.5%, current geometry students averaged 84.5%, and current precalculus students averaged 84.1%, all statistically equal to each other and all in the B grade range. Similarly, for the Algebra 1 assessment, current geometry students averaged 65.6%, statistically equal to the 65.5% of the precalculus students and in the D grade range. These data suggest that the mere passage of time does not degrade math skills, at least for the first few years.

Third, and perhaps the most interesting, degradation of math skills seemed to depend on the difficulty of the subject matter, with difficult math topics degrading more rapidly than easier ones. This can be shown by examining overall trends. Sixth graders who took the 5th grade math test, geometry students who took the Algebra 1 test and precalculus students who took the Algebra 2 test all took tests based on math they had completed the previous semester. Therefore, an equal amount of time had passed between learning and testing. Nevertheless, the mean score on the 5th grade test for 6th graders was 81.5, on the Algebra 1 test for geometry students was 65.6 and on the Algebra 2 test for precalculus students was 46.6. Given that students were spaced roughly 2-3 grades apart across each of these tests, it is reasonable to investigate whether there is a linear trend in the decline of these mean test scores. Accordingly, a Cochran-Armitage Test for Trend in Proportions was performed on these means to determine if there was a linear trend. The results of the analysis indicates there is, $z = 2.42$, $p < .05$, suggesting that the amount of degradation in math skills that occurred in the semester immediately after the course material was learned depended on difficulty.

A strong test of this hypothesis can be found by looking at the performance of the precalculus students on all three assessments. If degradation of math skills depends on time, one would expect performance to be strongest on the most recently learned material and weakest on the oldest material. If degradation of math skills depends on difficulty of the material, one would expect performance to be strongest on the oldest (easiest) material and weakest on the newest (most difficult) material. For precalculus students, the mean scores were 84.1% on the 5th grade math assessment, 65.5% on the Algebra 1 assessment and 46.6% on the Algebra 2 assessment. This trend is also consistent with the hypothesis that degradation depends on difficulty rather than passage of time and is statistically significant using the Cochran-Armitage Test for Trend in Proportions, $z = 2.16$, $p < .05$. This finding is perhaps even more significant given that the most

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recently-learned and most difficult material showed the lowest performance, thus supporting the idea that subject matter difficulty rather than passage of time is primarily responsible for degradation of performance.

Discussion

There were three main findings from the present research. First, there was significant degradation in performance across grade levels, particularly for the geometry and precalculus students. The decrease in performance was especially strong for Algebra 1 and Algebra 2 skills and strongest for the latter. The decrease in performance was much greater than what had previously been reported in the literature for students of similar grade levels.

Second, passage of time had virtually no effect on performance levels as students of different grade levels performed virtually identically when taking the same grade-level test. Third, degradation of performance depended on difficulty of the material as evidenced by the linear trend in performance when looking at student performance across grades for the previous grade's test or when looking at the performance of students within a grade when they took the different tests from different grade levels. This goes against the findings of Hill et al. (2007), which showed greater degradation of skills at younger ages (and presumably easier material).

The conclusion from the present study is that there appears to be significant degradation of mathematics skills and that this degradation appears to be due to the difficulty of the material rather than the amount of time that has elapsed between the time the material was learned to the time the material was assessed in the study.

One potential explanation for why older concepts had higher performance scores is that math is cumulative and, therefore, students used these concepts repeatedly over time, which reinforced the learning and prevented degradation of performance. This might explain why geometry students scored higher on 5th grade math topics than they did on Algebra 1 topics and why pre-calculus students scored higher on 5th grade math topics than they did on Algebra 1 topics, which, in turn, showed higher scores than Algebra 2 concepts. Under this explanation, precalculus students did worse on Algebra 2 topics than they did on Algebra 1 topics because they had less time to reinforce the learning of the former topics in their subsequent education.

The challenge with this explanation is two-fold. Under this explanation, we would expect to see higher performance for a given grade level test by older students since they had more time to reinforce their understanding of the earlier grade topics. This was decidedly not true as students of different grade levels scored virtually identically when taking the same test, suggesting there was no enhancement of knowledge through subsequent exposure. Moreover, there could be no

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ceiling effect argument invoked to explain why students did not do better as they progressed in math since scores in the 80's and 60's for 5th grade math and Algebra 1, respectively, are hardly near perfection.

A related explanation for why students of different grade levels scored virtually identically on the same grade level tests is that the forgetting that normally occurs with the passage of time was offset by the continued exposure to the concepts over the years. If this were true, then we would expect to see relatively constant performance across tests for the geometry and precalculus students. However, for students at both course levels, there was a virtually identical decline in performance as tests progressed from older material to newer material.

Another explanation for why students performed so poorly on the advanced topics is that they never learned them in the first place. This would be a plausible explanation if not for the dramatic drop in performance levels and the consistency of this drop across different age groups, who presumably learned the topics at different times in different classrooms. While it may be plausible to argue that the average 5th grade performance is in the 80's or B range, it does not make sense that B students become, on average, D students when they reach Algebra 1 and F students when they reach Algebra 2. By and large, the majority of the students who participated in the present study came from top-performing school districts.

Conclusion

Given that the present study indicates that math skills degrade rapidly very quickly and that this degradation is more severe for more complex material, the question becomes how can one prevent such skill decline from happening? One solution, which is suggested by the literature is overlearning. Overlearning refers to the process of receiving more instruction than is required to master the subject matter. Melnick (1971), for example, found that students who had received twice the amount of instruction necessary to achieve mastery of a topic performed better after four weeks than those simply trained to mastery level. The challenge with adopting this in a school setting is the limitations in class time available.

Another solution is to continue to review the subject matter. Teachers routinely begin the school year by reviewing key concepts from the previous year. Textbooks often begin with review of foundational concepts. This method may indeed be effective, and the present study suggests that as topics get more advanced, the frequency of required review periods will increase since skills associated with complex topics appear to degrade greatly in a short period of time.

An interesting question, and one worthy of further research, is whether there are ways to initially teach advanced topics such that the degradation curve is greatly slowed. Perhaps, we can look to

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Albert Einstein for inspiration on that question. He famously stated that given the postulates of geometry, he could see how the theorems would follow. In other words, rather than memorizing the theorems, Einstein could intuitively derive them given the foundational knowledge. Our results suggest that students are stronger on foundational concepts than they are on advanced concepts. Perhaps, teaching students the conceptual relationships between the foundational and the advanced concepts is a way to help students reconstruct the advanced concepts, so that even when they start forgetting them, they are able to derive them. A MyEdMaster study (Wang, Ailneni and Leddo, 2021) found that the inclusion of instruction of the concepts behind a mathematical procedure greatly increases problem solving performance compared to procedural instruction alone. Perhaps such a method could aid in the long term retention of these concepts.

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